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ON THE BOUNDS FOR THE STRESS INTENSITY FACTOR IN LAMINATED COMPOSITES*

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1. Introduction. In this note we consider the analysis of a plate which consists of parallel load-carrying strips of high modulus separated by buffer strips of relatively low modulus [1]. Both strips are boron-epoxy composite. Through an appropriate orientation of the fibers, a relatively low modulus and high toughness is achieved in the buffer strips. This in turn improves the crack arrest characteristics of the structure as a whole. Even though the primary (load-carrying) laminates and the buffer strips are highly anisotropic, in this note for reasons of analytical expediency, it will be assumed that both materials are isotropic and linearly elastic (Figure 1). A symmetrically located through crack is assumed to form and propagate in the primary laminate, and eventually to enter into the buffer strips. The objective of this note is to calculate an upper and a lower bound for the stress intensity factor when the crack tips are in the primary laminate, in the buffer strips, and at the interfaces.

2. Bounds for the stress intensity factor. The actual problem is described in Figure 1. The composite plate is assumed to contain a symmetrically located through crack of length $2a$,

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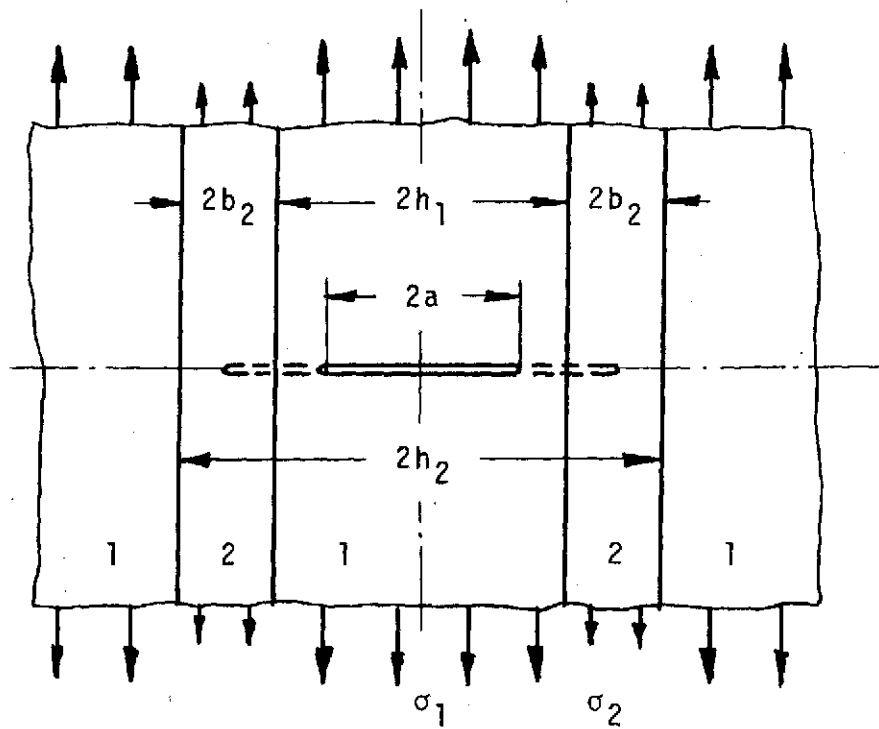


Figure 1. The geometry of the actual composite plate.

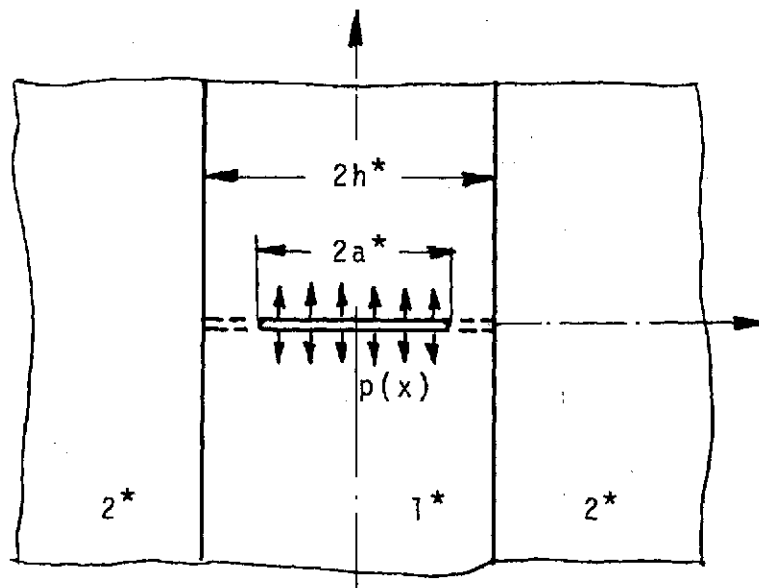


Figure 2. The geometry of idealized composite plate.

where $0 < a \leq h_2$, $h_2 = h_1 + 2b_2$, $2h_1$ and $2b_2$ being the widths of the primary laminate (material 1) and the buffer strip (material 2), respectively. The crack is perpendicular to the bimaterial interfaces. The plate is subjected to uniaxial loading parallel to the strips outside the perturbation zone of the crack. The elastic constants of the materials 1 and 2 are E_1 , ν_1 (or $\mu_1 = E_1/2(1+\nu_1)$, $\kappa_1 = (3-\nu_1)/(1+\nu_1)$) and E_2 , ν_2 (or μ_2 , κ_2), respectively. The elastic constants of the composite plate may be evaluated from

$$\begin{aligned} E_c &= (h_1 E_1 + b_2 E_2)/(h_1 + b_2) , \\ \nu_c &= (h_1 \nu_1 + b_2 \nu_2)/(h_1 + b_2) . \end{aligned} \quad (1)$$

In the plate without the crack the stresses σ_1 and σ_2 acting on the strips in a plane perpendicular to the interfaces are related by

$$\sigma_1/\sigma_2 = E_1/E_2 . \quad (2)$$

The solution of the problem described in Figure 1 is not available and, at present, appears to be intractable. However, through a judicious choice of the elastic constants and the dimensions in the problem described in Figure 2 (the solution of which is available [2]) one could find estimates for an upper and a lower bound for the corresponding stress intensity factors. One simple procedure to obtain these estimates is the following:

(a) An upper bound for the stress intensity factor k when the crack is in the primary laminate: $0 < a \leq h_1$.

This estimate may easily be obtained by replacing the

part of the composite plate $|x| > h_2$ (see Figure 1) by homogeneous half planes having the elastic constants of the buffer strip, E_2, ν_2 . Since the stiffness of the resulting half planes $|x| > h_1$ is less than that of the half planes in the actual structure, the stress intensity factor calculated from Figure 2 with

$$\begin{aligned} p(x) &= -\sigma_1, & a^* &= a \leq h_1, & h^* &= h_1, & E_1^* &= E_1, \\ \nu_1^* &= \nu_1, & E_2^* &= E_2, & \nu_2^* &= \nu_2, \end{aligned} \quad (3)$$

would be greater than the k value corresponding to Figure 1.

(b) A lower bound for k when the crack is in the primary laminate: $0 < a \leq h_1$.

This estimate may be obtained by replacing the non-homogeneous composite plate for $|x| > h_1$ by the equivalent homogeneous composite half planes where

$$\begin{aligned} p(x) &= -\sigma_1, & a^* &= a \leq h_1, & h^* &= h_1, & E_1^* &= E_1, \\ \nu_1^* &= \nu_1, & E_2^* &= E_c, & \nu_2^* &= \nu_c. \end{aligned} \quad (4)$$

Here the reason for the resulting k being lower than the actual k is that the stiffness E_c used for calculations in Figure 2 is greater than E_2 which is the stiffness of the neighboring media in the actual problem (Figure 1).

(c) A lower bound for k when the crack is in the buffer strip: $h_1 < a \leq h_2$.

This estimate may be obtained by replacing the nonhomogeneous composite plate by a homogeneous strip (E_2, ν_2) (which has a stiffness smaller than the actual) for $|x| < h_2$ and by

homogeneous half planes (E_1, ν_1) (which have a stiffness greater than the actual) for $|x| > h_2$ and using the pressure $p = -\sigma_2$ as the external load. Thus in Figure 2 the parameters would be

$$\begin{aligned} p(x) &= -\sigma_2, & a^* &= a, & h^* &= h_2, & E_1^* &= E_2, \\ \nu_1^* &= \nu_2, & E_2^* &= E_1, & \nu_2^* &= \nu_1. \end{aligned} \quad (5)$$

(d) A closer lower bound for k when the crack is in the buffer strip: $h_1 < a < h_2$.

This estimate is obtained by using the same plate as in (c) with the actual (and higher) crack surface pressure, i.e.,

$$\begin{aligned} a^* &= a, & h^* &= h_2, & E_1^* &= E_2, & \nu_1^* &= \nu_2, & E_2^* &= E_1, \\ \nu_2^* &= \nu_1, & p(x) &= \begin{cases} -\sigma_1 & \text{for } |x| < h_1, \\ -\sigma_2 & \text{for } h_1 < |x| < a. \end{cases} \end{aligned} \quad (6)$$

Theoretically, the k value given by this estimate is still a lower bound, but is believed to be quite close to the actual stress intensity factor.

In the problems described above the solution is obtained through a superposition where the stress intensity factor is calculated from the singular solution in which the crack surface pressure is the only external load (Figure 2). In the solution given in this note, as long as the crack tip is away from the interfaces, the following asymptotic standard expression is valid for the cleavage stress σ_{yy} around the crack tip:

$$\sigma_{yy}(x,0) \approx \frac{k}{\sqrt{2(x-a)}}, \quad (a \neq h_i, i=1,2). \quad (7)$$

However, if the crack tip terminates at an interface, the asymptotic stresses around the crack tip are given by [3,4]

$$\sigma_{ij}(r, \theta) = \frac{k}{\sqrt{2}} r^\alpha f_{ij}(\theta) + O(r^{\alpha+2}), \quad (i, j = r, \theta; \quad \text{Re}(\alpha_2) > 0, \quad 0 < \alpha < 1), \quad (8)$$

where the crack is assumed to lie along $\theta = \pi$ (i.e., the negative x axis) [3,4]. Following Figure 3, if $\mu_2 > \mu_1$ then $0 < \alpha < 0.5$ and if $\mu_2 < \mu_1$ then $0.5 < \alpha < 1$. The functions $f_{ij}(\theta)$ giving the angular variation of the stresses around the crack tip for a symmetrically loaded crack (i.e., first mode) and for various values of μ_2/μ_1 are shown in Figures 3-5 in the following normalized form:

$$f_{ij}(\theta) = G_{ij}(\theta)/G_{\theta\theta}(0), \quad (i, j = r, \theta). \quad (9)$$

3. The Numerical Results. The numerical results given in this section are obtained for the boron-epoxy composite plate which was considered in [1]. The material constants of the (equivalent) isotropic materials are:

Primary laminates: $E_1 = 11.39 \times 10^6 \text{ psi}$, $\nu_1 = 0.33$,

Buffer strips: $E_2 = 2.42 \times 10^6 \text{ psi}$, $\nu_2 = 0.85$,

Composite plate ($h_1/b_1 = 2$): $E_c = 8.4 \times 10^6 \text{ psi}$, $\nu_c = 0.503$,

(i.e., $\mu_1/\mu_2 = 6.55$, $\kappa_1 = 2$, $\kappa_2 = 1.16$, $\mu_c/\mu_1 = 0.657$,

$\kappa_c = 1.66$).

The results corresponding to the cases (a)-(d) described in Section 2 are given in Tables 1-4, respectively, and are summarized in Figure 6. Note that Table 1 is valid for any value of

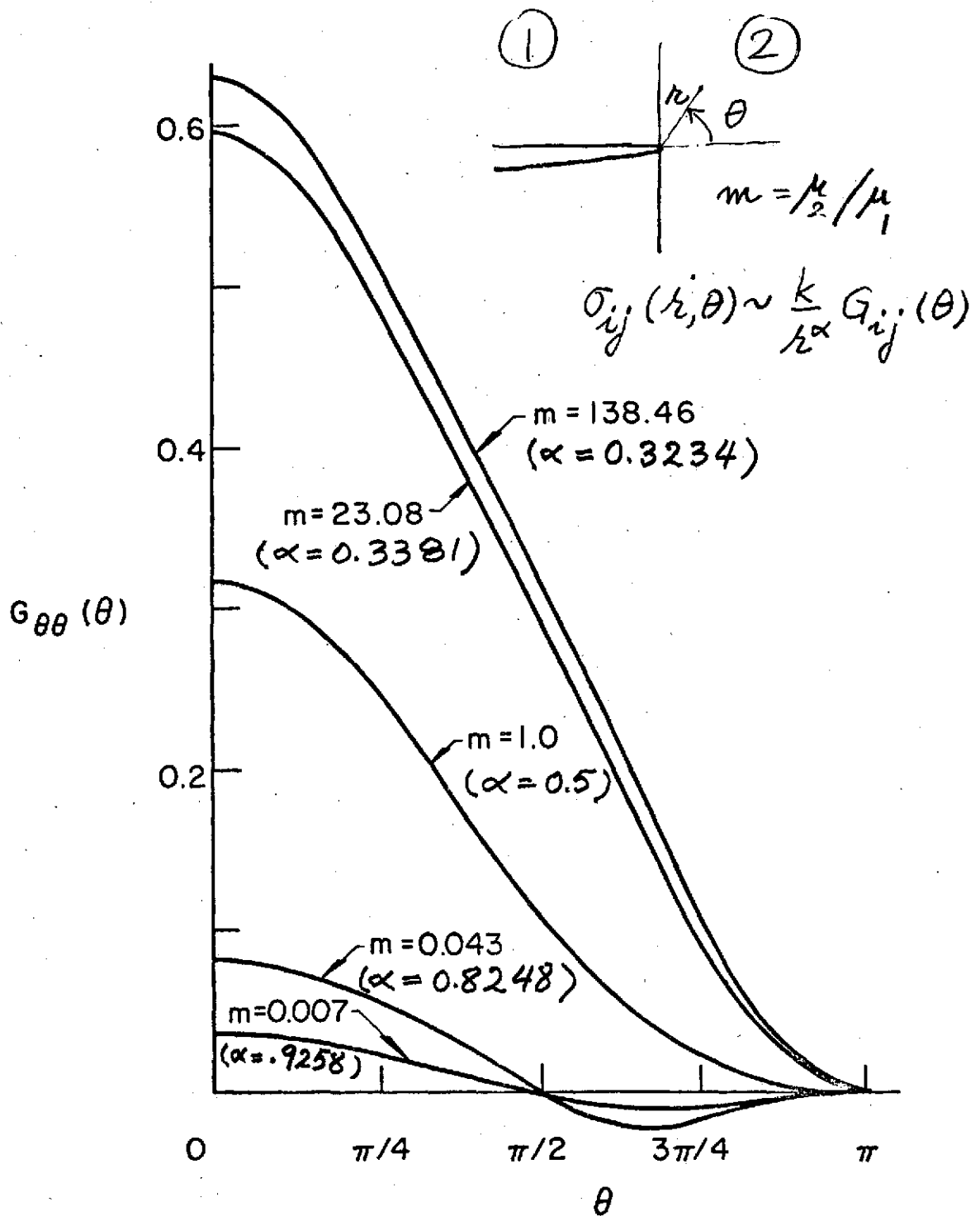


Figure 3. Angular variation of $\sigma_{\theta\theta}$ around a crack tip touching the interface.

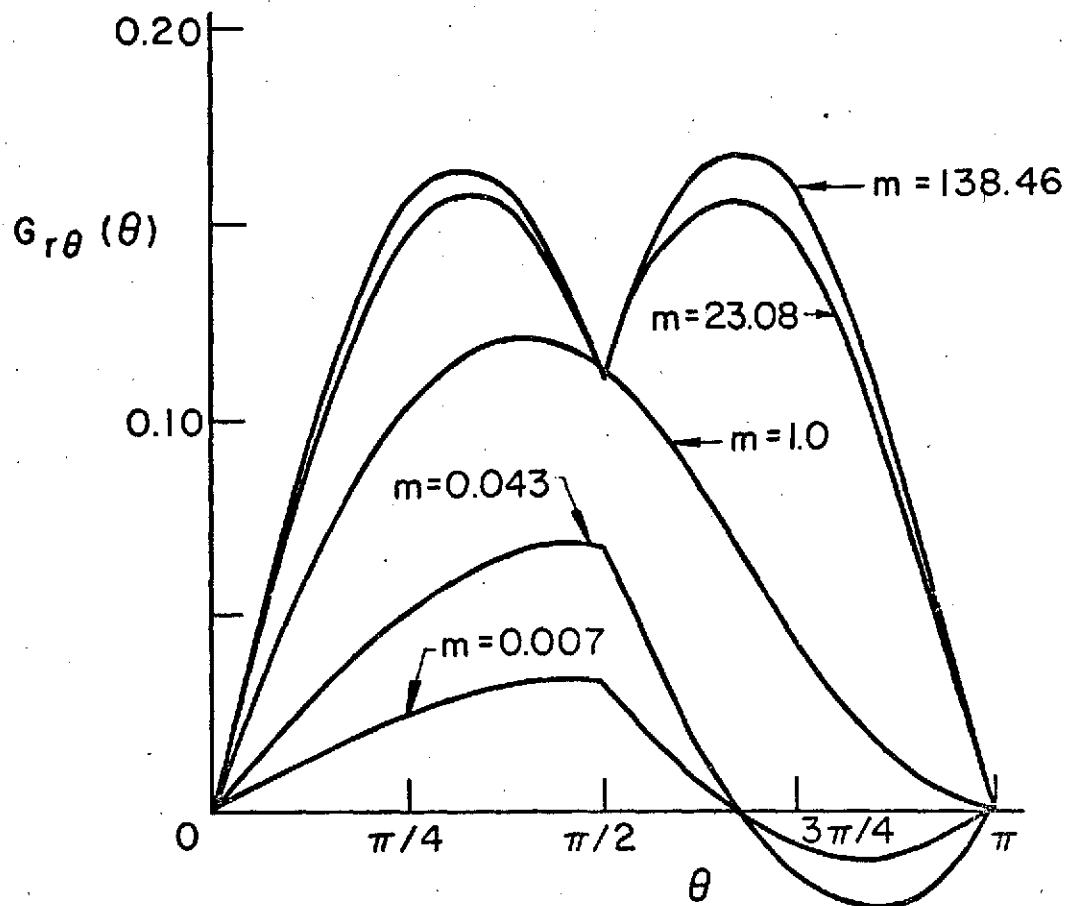


Figure 4. Angular variation of $\sigma_{r\theta}$ around a crack tip touching the interface.

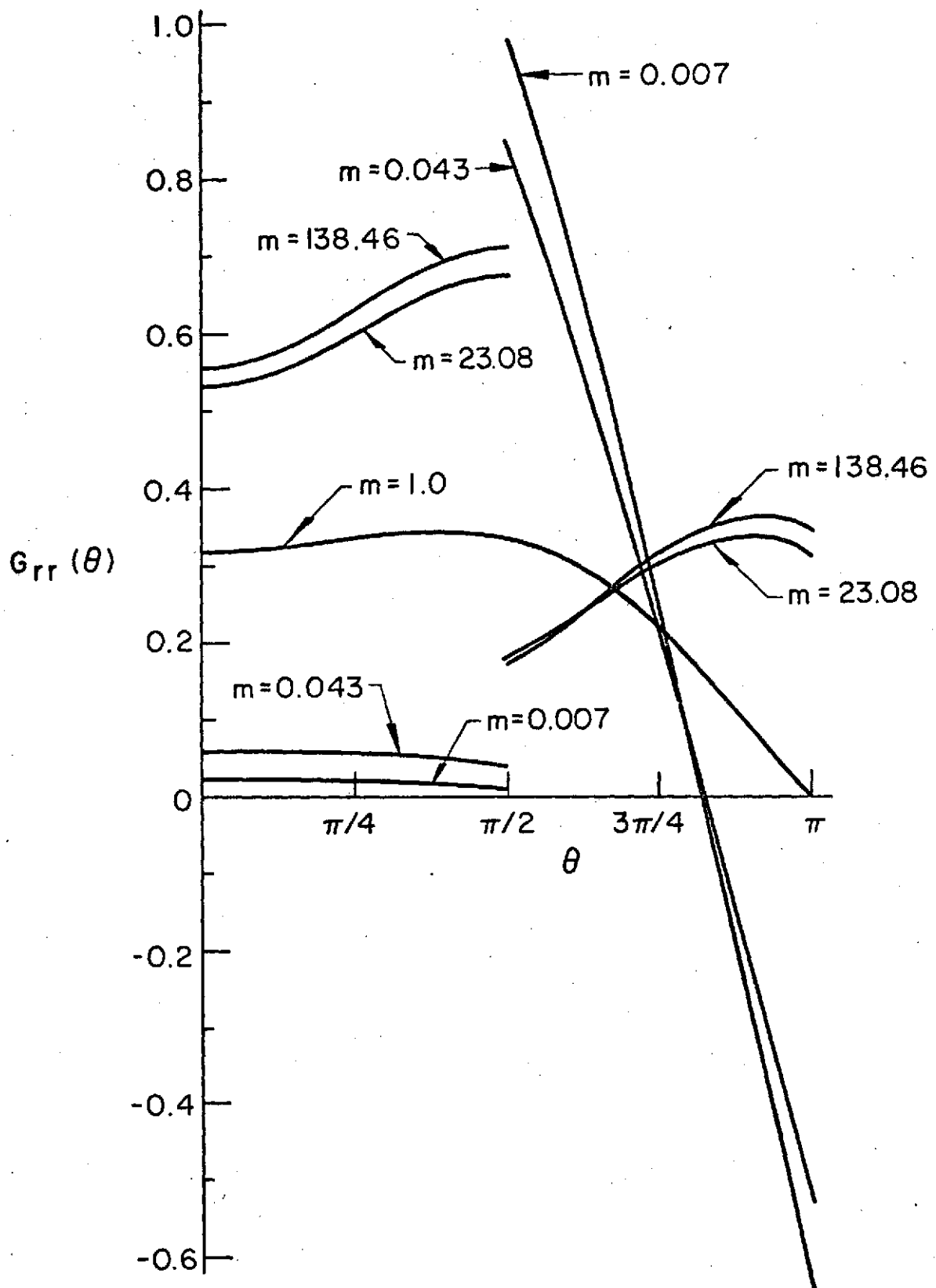


Figure 5. Angular variation of σ_{rr} around a crack tip touching the interface.

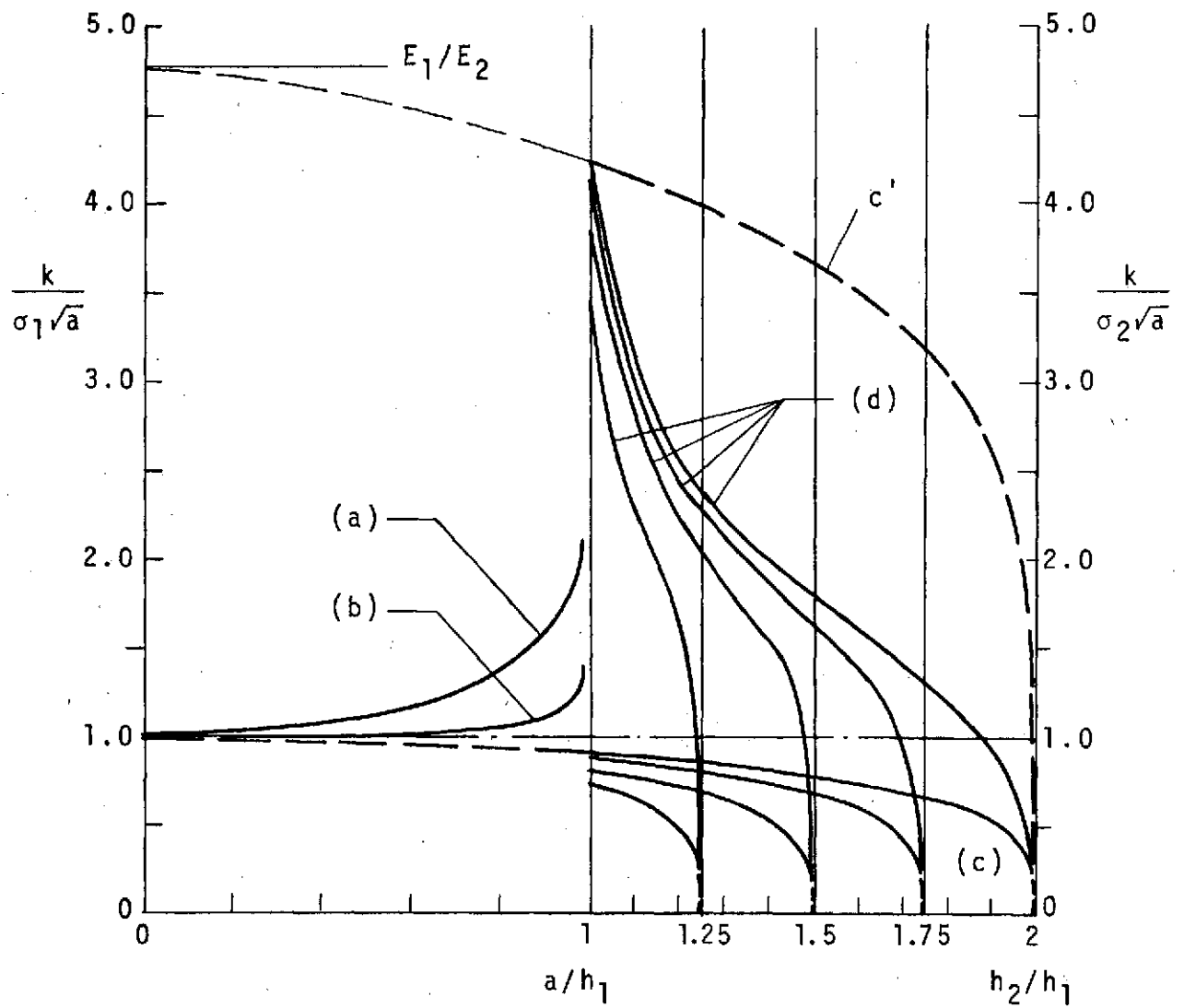


Figure 6. Bounds for the stress intensity factor in the composite plate.

the width ratio h_1/b_2 and for $0 < a \leq h_1$. Even though the results given in Table 3 are obtained only for $h_2/h_1 = 2$, it is clear that since the pressure on the crack surface is uniform, the results for any other value of h_2/h_1 can be obtained from those given for $h_2/h_1 = 2$ by using appropriate a/h_2 ratios. Thus Figure 6 shows the curves for four different values of h_2/h_1 ratio, namely 2, 1.75, 1.5, and 1.25.

Table 1. Upper bound for the stress intensity factor: $0 < a \leq h_1$, (crack in primary laminate), $\kappa_1 = 2$, $\kappa_2 = 1.16$, $\mu_2/\mu_1 = 0.1527$, uniform crack surface pressure, $-\sigma_1$.

a/h_1	0.14	0.225	0.325	0.6	0.75	0.9	1.0
$k/\sigma_1\sqrt{a}$	1.007	1.018	1.038	1.157	1.296	1.618	(0.550) $\alpha = 0.66980$

Table 2. Lower bound for k : $0 < a \leq h_1$, $\kappa_1 = 2$, $\kappa_2 = \kappa_c = 1.66$, $\mu_c/\mu_1 = 0.657$, $h_1/b_2 = 2$, (crack in primary laminate), uniform crack surface pressure, $-\sigma_1$.

a/h_1	0.6	0.75	0.9	1.0
$k/\sigma_1\sqrt{a}$	1.031	1.055	1.099	(0.920) $\alpha = 0.52620$

Table 3. Lower bound for k : $h_2 \geq a > h_1$, (crack in buffer strip), $\kappa_1 = 2$, $\kappa_2 = 1.16$, $\mu_1/\mu_2 = 6.55$, uniform crack surface pressure, $-\sigma_2$.

a/h_2	0.5	0.75	0.875	1.0
$k/\sigma_2\sqrt{a}$	0.899	0.772	0.675	(1.160) $\alpha = 0.42724$

Table 4. A (closer) lower bound for k : $h_2 \geq a > h_1$ (crack in buffer strip), $\kappa_1 = 2$, $\kappa_2 = 1.16$, $\mu_1/\mu_2 = 6.55$, crack surface pressure $p(x) = -\sigma_1$ for $0 \leq |x| \leq h_1$, $p(x) = -\sigma_2$ for $h_1 < |x| < a$, $p_2/p_1 = E_2/E_1$.

$\frac{h_1}{h_2} = \frac{1}{2}$	a/h_2	0.5	0.625	0.75	0.875	1.0
	$k/\sigma_2\sqrt{a}$	4.231	2.396	1.820	1.302	(2.221) $\alpha = 0.42724$
$\frac{h_1}{h_2} = \frac{1}{1.75}$	a/h_2	1/1.75	0.68	0.92	1.0	
	$k/\sigma_2\sqrt{a}$	4.120	2.580	1.397	(2.200) $\alpha = 0.42724$	
$\frac{h_1}{h_2} = \frac{1}{1.5}$	a/h_2	2/3	0.8	0.94	1.0	
	$k/\sigma_2\sqrt{a}$	3.85	2.257	1.536	(2.900) $\alpha = 0.42724$	
$\frac{h_1}{h_2} = \frac{1}{1.25}$	a/h_2	0.8	0.88	0.96	1.0	
	$k/\sigma_2\sqrt{a}$	3.43	2.286	1.666	(3.650) $\alpha = 0.42724$	

Table 4 shows the results corresponding to the assumption (d) of the previous section for four different values of h_2/h_1 . It should again be noted that these results give a (closer) lower bound for k when the crack is in the buffer strip. Since in this case changing the material constants to obtain an upper bound is not realistic, an approximation to an upper bound may be obtained by changing the crack surface tractions. This may be done by using the highest possible uniform crack surface pressure, $p = -\sigma_1$ instead of the lowest uniform pressure $p = -\sigma_2$ in the calculations. The result would be simply the multiplication of the values found in Table 3 and Figure 6 (c) by the factor $\sigma_1/\sigma_2 = E_1/E_2 = 4.7066$

(see, for example, the dashed curve (c') in Figure 6 which corresponds to $h_2/h_1 = 2$). These results are repeated in Table 5.

Table 5. An (approximate) upper bound for k : $h_2 \geq a > h_1$ (crack in buffer strip), $\kappa_1 = 2$, $\kappa_2 = 1.16$, $\mu_1/\mu_2 = 6.55$, $p(x) = -\sigma_1$, ($0 \leq |x| < a$).

a/h_2	0.5	0.75	0.875	1.0
$k/\sigma_2\sqrt{a}$	4.231	3.634	3.177	(5.460) $\alpha = 0.42724$

The tables 1-5 also give the results for the crack terminating at a bi-material interface (i.e., $a = h_1$ or $a = h_2$). The corresponding k values shown in the tables in parentheses and the exponents α should be understood (and used) in the sense as defined by eq. (8). Note that the k values given in the tables and in Figure 6 tend to infinity or zero as the crack tip approaches the interface. The reason for this, of course, is found in the definition of k as given by eq. (7) and in the abrupt change in the power of the singularity α when the crack tip touches the interface. A possible technique of applying these results regarding the irregular singularity $\alpha \neq 0.5$ in fracture studies may be found in [5].

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